L. Hamilton June 28, 2012

July Report based on June Data

PAN-ARCTIC OUTLOOK — Hamilton

1. Extent Projection

A Gompertz (asymmetrical S curve) model estimated by iterative least squares suggests a mean September 2012 ice extent of **4.3 million km^2** (NSIDC). *The 95% confidence interval for this prediction ranges from 3.4 to 5.1 million km^2.*

2. Methods / Techniques

Figure 1 shows this naive, purely statistical model. It predicts September mean extent from a Gompertz curve representing the trend over previous years. Estimation data are the **NSIDC monthly mean extent** reports from September 1979 through September 2011.

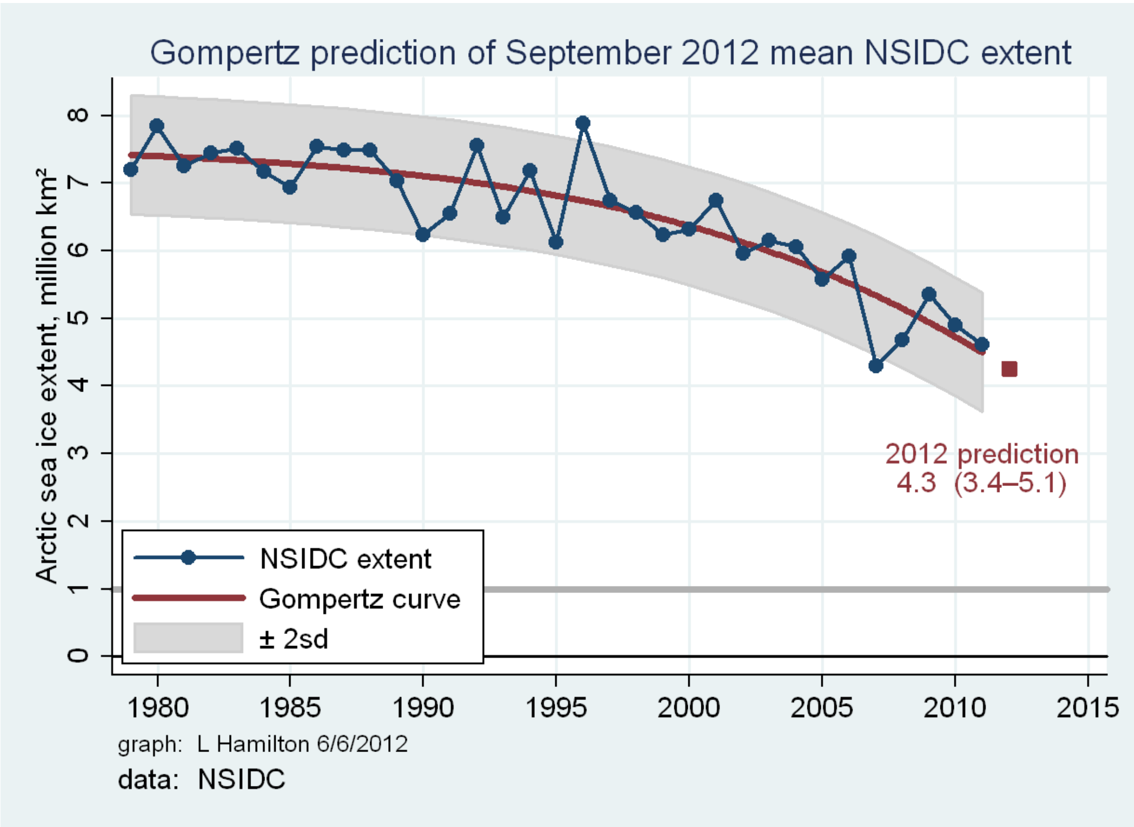

Figure 1

Parameters for the model are estimated via iterative least squares, using the **nl** procedure of Stata 12.1. Figure 1 also shows confidence bands calculated as the prediction plus or minus twice the standard deviation of the residuals.

In the command below, **gom3** specifies a 3-parameter Gompertz curve. ***extent*** refers to September mean NSIDC sea ice extent, in millions of km^2. ***year*** refers to the calendar year.

. use C:\data\Arctic9.dta, clear

```
(Arctic September mean sea ice 1979-2011)
```

. nl gom3: *extent year*, nolog

```
(obs = 33)

      Source |       SS       df       MS
-------------+------------------------------         Number of obs =        33
       Model |  1425.43798     3  475.145994         R-squared     =    0.9957
    Residual |  6.15941312    30  .205313771         Adj R-squared =    0.9953
-------------+------------------------------         Root MSE      =  .4531156
       Total |   1431.5974    33  43.3817393         Res. dev.     =  38.25858

3-parameter Gompertz function, extent = b1*exp(-exp(-b2*(year - b3)))
------------------------------------------------------------------------------
      extent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         /b1 |   7.580278    .291652    25.99   0.000     6.984645    8.175911
         /b2 |  -.0995915   .0271646    -3.67   0.001     -.155069    -.044114
         /b3 |   2017.531   2.173212   928.36   0.000     2013.093    2021.969
------------------------------------------------------------------------------
```

The squared correlation between observed and predicted values (not shown) is $r^2 = .79$. There is no significant autocorrelation among the residuals, as tested by Ljung–Box *Q* statistics.

. predict *resid*, resid
. corrgram *resid*, lag(6)

```
                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1      -0.2135  -0.2138   1.6449  0.1996          -|                 -|
2      -0.0599  -0.1094   1.7787  0.4109           |                  |
3      -0.2277  -0.3000   3.7748  0.2868          -|                --|
4       0.0233  -0.1280   3.7963  0.4343           |                 -|
5       0.1641   0.1840   4.9073  0.4273           |-                 |-
6      -0.1815  -0.2761   6.3169  0.3886          -|                --|
```

3. Rationale

This is a naive model proposed before the start of the 2012 melt season. Most trend-line analyses of Arctic sea ice have used linear, quadratic, exponential or logistic models. The Gompertz curve appears preferable to these alternatives in several respects.

- It follows the observed pattern of gradually accelerating decline in the 1970s and 80s.
- The decline later steepens at an accelerating rate, as observed since the mid-2000s.
- Model predictions do not cross or exactly reach zero extent. Rather they approach this limit asymptotically.
- The asymmetrical-S shape bears a qualitative resemblance to results from much more elaborate physical models, such as those reported by the IPCC (2007); other sigmoid projections in Holland et al. (2006), Wang & Overland (2009), or Boe, Hall & Qu (2009).

Although out-of-sample extrapolation of this non-physical model is purely speculative, it is interesting to note the suggestion of extent falling below 1 million km^2 by 2025. More realistically, if we add Gaussian noise with the same standard deviation as past residuals to the projected future curve, we see behavior like the four examples in Figure 2.

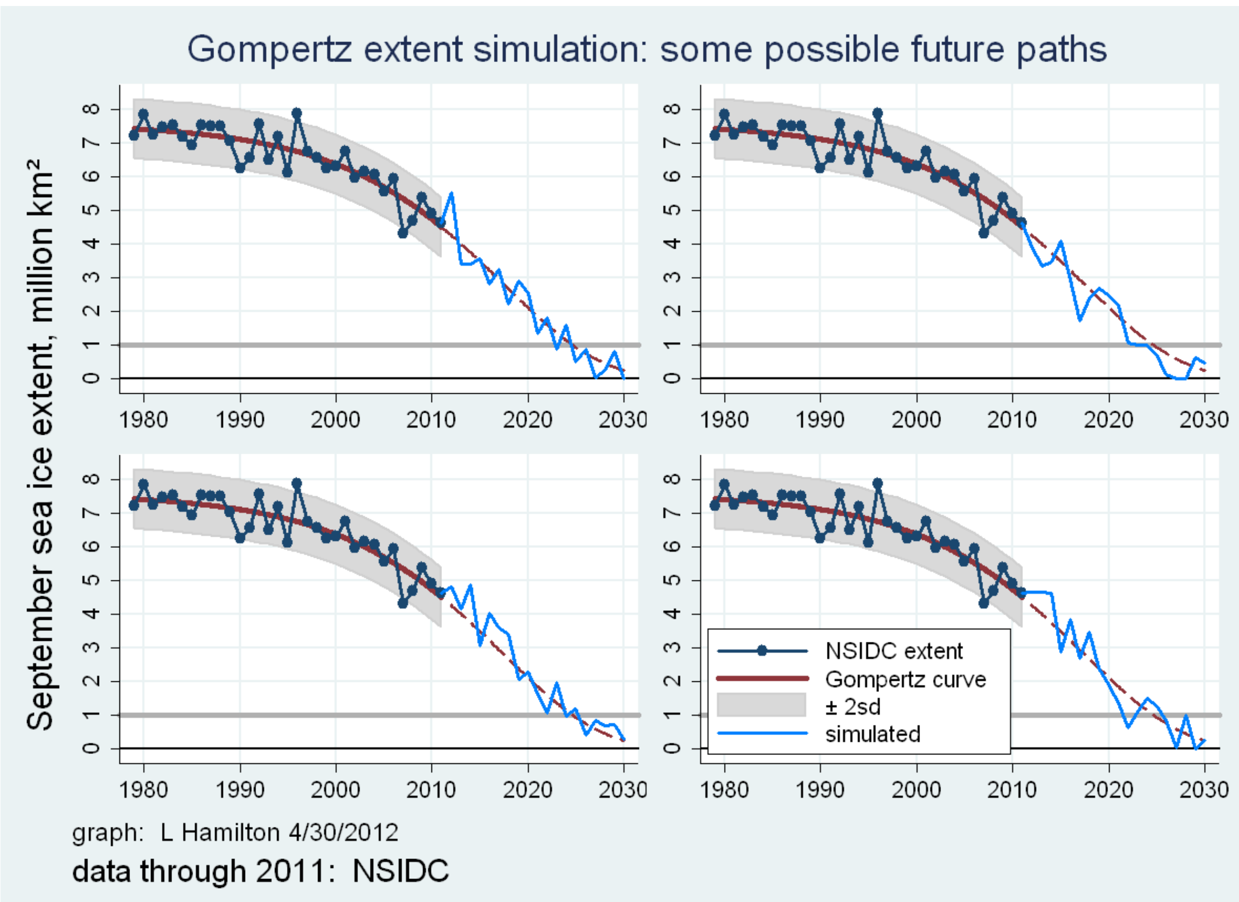

Figure 2

Continuing in this vein, the same curve-fitting approach yields a September 2012 mean **NSIDC area** prediction of 3 million km^2, with confidence interval from 2.2 to 3.7 (Figure 3).

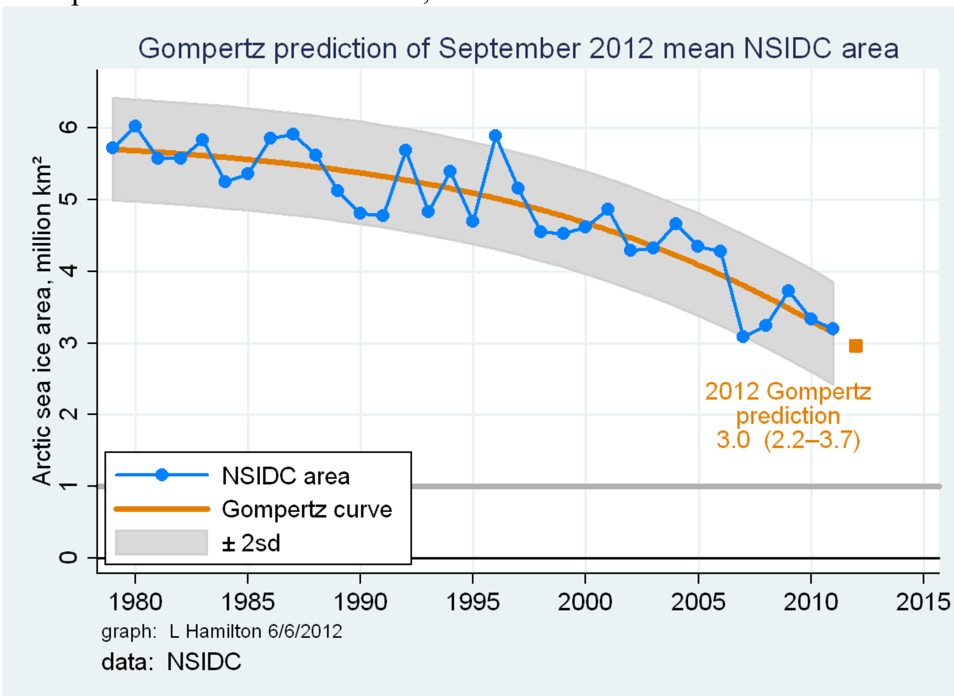

Figure 3

The **PIOMAS volume** prediction is 4 thousand km^3, with confidence interval from 2 to 5.9 (Figure 4).

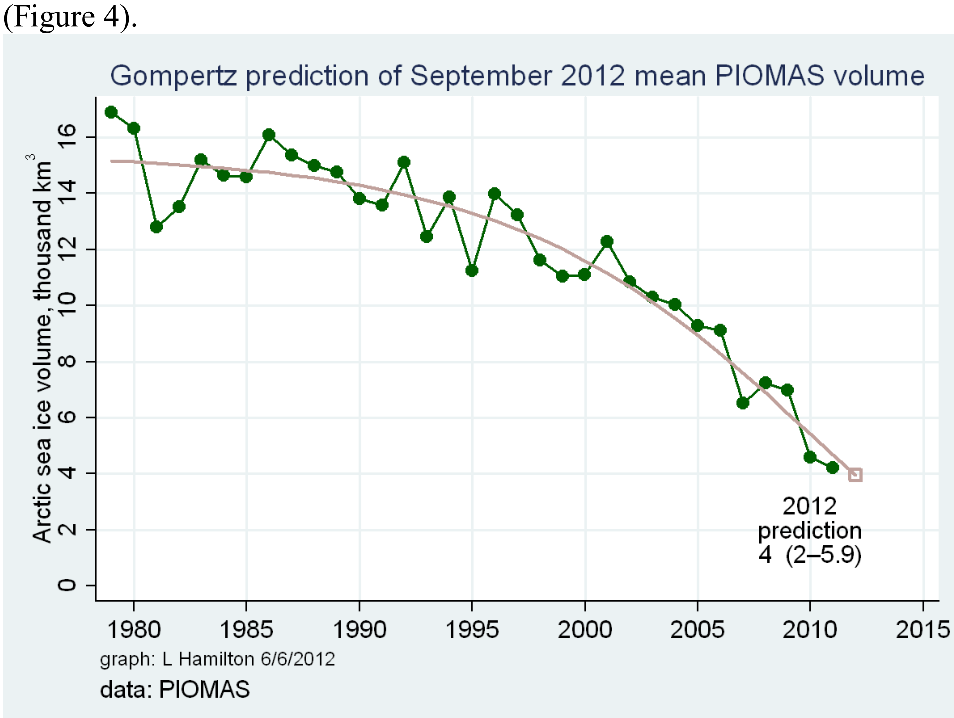

Figure 4

But all these are monthly means. What about daily values? They are often disdained too random to reasonably predict. I tried this out anyway, coming up with a minimum 1-day **Cryosphere Today extent** prediction of 2.7 million km^2, with confidence interval from 2.1 to 3.3 (Figure 4).

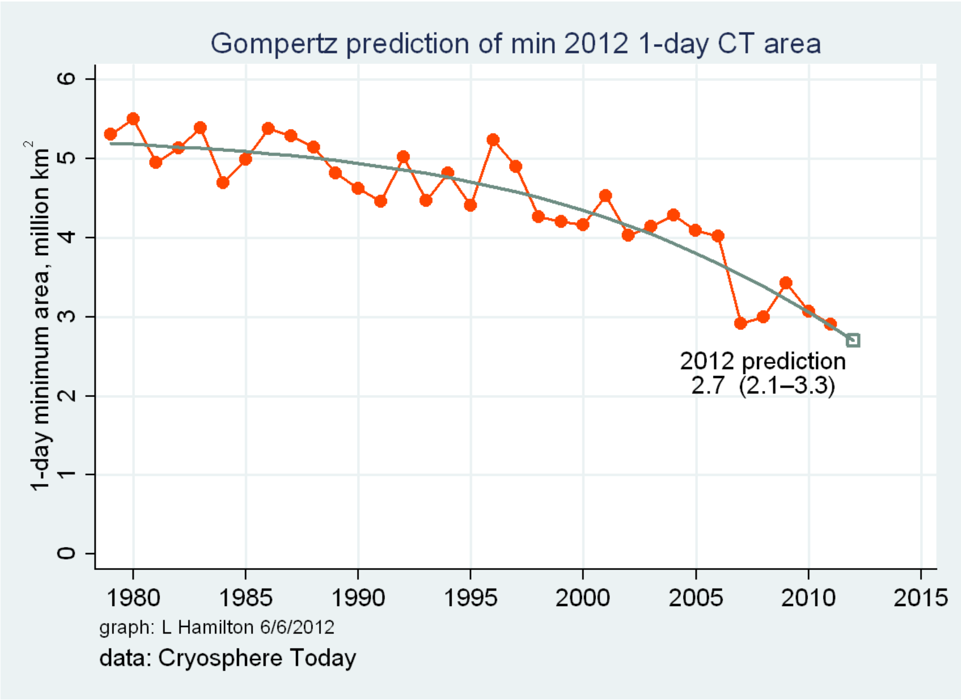

Figure 5

Are minimum daily values really more difficult to predict? The surprising answer is no, at least with respect to Cryosphere Today area. The Gompertz model for minimum daily area graphed above fits with a squared correlation of .984, and a residual standard deviation of .292 million km^2. A very similar model for CT September mean has a slightly lower r^2 (.95) and higher residual sd (.343).